MA114 Summer II 2018
Worksheet 1b

1. Compute the integrals below:

a) $\int \arcsin(x)\,dx$

Hint: Remember that the derivative of $\arcsin(x)$ is $\frac{1}{\sqrt{1-x^2}}$ and emulate the $\arctan(x)$ example from class.

$u = \arcsin(x)$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

$\int \arcsin(x)\,dx = x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx$

Substitute $t = 1-x^2$, $dt = -2x\,dx \Rightarrow dx = \frac{dt}{-2x}$

$= x\arcsin(x) - \int \frac{\cancel{x}}{\sqrt{t}}\,\frac{dt}{-2\cancel{x}}$

$= x\arcsin(x) + \int \frac{1}{2\sqrt{t}}\,dt$

$= x\arcsin(x) + \sqrt{t} + C = \boxed{x\arcsin(x) + \sqrt{1-x^2} + C}$

b) $\int e^{2x}\sin(x)\,dx$,

$u = e^{2x}$ $\quad dv = \sin(x)\,dx$

$du = 2e^{2x}dx$ $\quad v = -\cos(x)$

$\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + \int 2e^{2x}\cos(x)\,dx$

$u = 2e^{2x}$ $\quad dv = \cos(x)\,dx$

$du = 4e^{2x}dx$ $\quad v = \sin(x)$

$= -e^{2x}\cos(x) + 2e^{2x}\sin(x) - 4\int e^{2x}\sin(x)\,dx$

$5\int e^{2x}\sin(x)\,dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x) + C$

$\boxed{\int e^{2x}\sin(x)\,dx = \frac{1}{5}\left(-e^{2x}\cos(x) + 2e^{2x}\sin(x)\right) + C}$

2. Let $f(x)$ be a twice differentiable function with $f(1) = 2, f(4) = 7, f'(1) = 5$, and $f'(4) = 3$. Evaluate $\int_1^4 xf''(x)\,dx$. Hint: What is $\int_1^4 f'(x)\,dx$?

Use by parts with $u = x$, $dv = f''(x)\,dx$, $du = dx$, $v = f'(x)$

$\int_1^4 xf''(x)\,dx = xf'(x)\Big|_1^4 - \int_1^4 f'(x)\,dx$

$= [4(3) - 1(5)] - f(x)\Big|_1^4 = 7 - [7-2] = \boxed{2}$

3. What is $\frac{1}{x} - \frac{1}{x+1}$?

$\frac{1}{x} - \frac{1}{x+1} = \frac{(x+1)}{x(x+1)} - \frac{x}{x(x+1)} = \boxed{\frac{1}{x^2+x}}$